Analysis of Guided-Wave Problems in Substrate Integrated Waveguides — Numerical Simulations and Experimental Results

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***Abstract* — In this paper, a novel finite difference frequency domain (FDFD) algorithm is proposed for the analysis of substrate integrated waveguide (SIW) guided-wave problems where the perfectly matched layer (PML) has been chosen as the absorbing boundary condition and the Floquet theorem has been used to consider the periodic structure. Using the above method, the guided-wave problem is then converted to a generalized matrix eigenvalue problem and finally transformed to a standard matrix eigenvalue problem, which can be easily solved with many efficient available subroutines. Excellent agreements of numerical simulations and experimental results show the validity and efficiency of the proposed algorithm.**

I. INTRODUCTION

In a microwave system, active devices in the form of chips are often surface-mounted on a planar carrier substrate, while the high-Q passive components like diplexers and filters are usually designed on the basis of rectangular waveguide or other non-planar structures. Because of the high cost and large size of the rectangular-waveguide components, the idea of integrated substrate rectangular waveguide (SIW) was proposed [1]-[4]. Thus, the microwave system can be made even in a package, by reducing the size, weight and cost, and also enhancing greatly the manufacturing repeatability and reliability. Figure 1 shows the structure of an SIW, which is synthesized with linear arrays of metallic via holes on a piece of low-loss substrate, for example, the low-temperature co-fired ceramics (LTCC). Recently, many researchers have been involved in the study of SIW [2-4].

In this paper, we develop a novel finite difference frequency domain (FDFD) algorithm to analyze propagation properties in an open periodic structure of SIW, where the perfectly matched layer (PML) has been chosen as the absorbing boundary condition (ABC) and the Floquet theorem has been employed. Using such an algorithm, a generalized matrix eigenvalue problem is obtained, which can be converted to an easily solved standard matrix eigenvalue problem. Numerical results have been compared with the measurement data in the frequency range of 26.5-40 GHz to show the validity and efficiency of the proposed algorithm.

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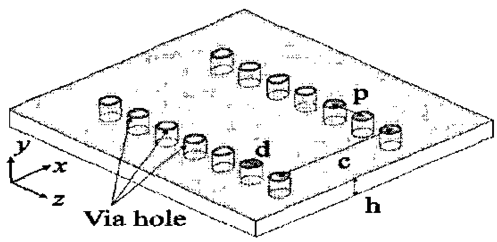

Fig. 1. The configuration of a SIW synthesized using via holes array.

II. FORMULATION AND NUMERICAL RESULTS

In a Cartesian coordinate system, the six components of the electric and magnetic fields yield 12 subcomponents in PML medium, and Maxwell's equations are replaced by the following 12 equations [6]:

$$\varepsilon \frac{\partial E'_{xy}}{\partial t} + \sigma_y E'_{xy} = \frac{\partial (H'_{zx} + H'_{zy})}{\partial y} \tag{1a}$$

$$\varepsilon \frac{\partial E'_{xz}}{\partial t} + \sigma_z E'_{xz} = -\frac{\partial (H'_{yz} + H'_{yx})}{\partial z} \tag{1b}$$

$$\varepsilon \frac{\partial E'_{yz}}{\partial t} + \sigma_z E'_{yz} = \frac{\partial (H'_{xy} + H'_{xz})}{\partial z} \tag{1c}$$

$$\varepsilon\frac{\partial E'_{yx}}{\partial t}+\sigma_x E'_{yx}=-\frac{\partial(H'_{zx}+H'_{zy})}{\partial x} \qquad (1d)$$

$$\varepsilon\frac{\partial E'_{zx}}{\partial t}+\sigma_x E'_{zx}=\frac{\partial(H'_{yz}+H'_{yx})}{\partial x} \qquad (1e)$$

$$\varepsilon\frac{\partial E'_{zy}}{\partial t}+\sigma_y E'_{zy}=-\frac{\partial(H'_{xy}+H'_{xz})}{\partial y} \qquad (1f)$$

$$\mu\frac{\partial H'_{xy}}{\partial t}+\sigma^*_y H'_{xy}=-\frac{\partial(E'_{zx}+E'_{zy})}{\partial y} \qquad (1g)$$

$$\mu\frac{\partial H'_{xz}}{\partial t}+\sigma^*_z H'_{xz}=\frac{\partial(E'_{yz}+E'_{yx})}{\partial z} \qquad (1h)$$

$$\mu\frac{\partial H'_{yz}}{\partial t}+\sigma^*_z H'_{yz}=-\frac{\partial(E'_{xy}+E'_{xz})}{\partial z} \qquad (1i)$$

$$\mu\frac{\partial H'_{yx}}{\partial t}+\sigma^*_x H'_{yx}=\frac{\partial(E'_{zx}+E'_{zy})}{\partial x} \qquad (1j)$$

$$\mu\frac{\partial H'_{zx}}{\partial t}+\sigma^*_x H'_{zx}=-\frac{\partial(E'_{yz}+E'_{yx})}{\partial x} \qquad (1k)$$

$$\mu\frac{\partial H'_{zy}}{\partial t}+\sigma^*_y H'_{zy}=\frac{\partial(E'_{xy}+E'_{xz})}{\partial y} \qquad (1l)$$

where

$$\begin{cases} E'_i = E'_{ij}+E'_{ik} & i,j,k\in\{x,y,z\} \\ H'_i = H'_{ij}+H'_{ik} & and(i\neq j; i\neq k; j\neq k) \end{cases} \qquad (2)$$

are components of the electric and magnetic fields, σ and σ^* satisfy the condition $\sigma/\sigma^* = \varepsilon/\mu$. We choose PML (4, 2, 10^{-4}) in the practical simulation [5].

From the Floquet theorem [6], one knows that there is no difference except a factor between two-group components whose locations are separated by a distance of the period in the stable electromagnetic fields. In a single period, each component of the electromagnetic fields may be expressed as

$$\begin{cases} \psi'(x,y,z,t)=\psi(x,y,z)e^{j\omega t-\gamma z} & \gamma=\alpha+j\beta \\ \psi(x,y,z+p)=\psi(x,y,z) & \alpha\beta\geq 0 \end{cases} \qquad (3)$$

where ψ' and ψ represent (E'_{ij}, H'_{ij}) and (E_{ij}, H_{ij}), respectively, and γ is the complex propagation constant.

Substituting (2) and (3) into (1) and eliminating E_{zx}, E_{zy}, H_{zx} and H_{zy}, we obtain the following set of equations [7]

$$\begin{cases} \gamma H_x = \frac{\partial H_x}{\partial z} - \frac{g_3}{g_1 g_2^*}\frac{\partial^2 E_x}{\partial x\partial y} - g_3 E_y + \frac{g_3}{g_1 g_1^*}\frac{\partial^2 E_y}{\partial x^2} \\ \gamma H_y = \frac{\partial H_y}{\partial z} + g_3 E_x - \frac{g_3}{g_2 g_2^*}\frac{\partial^2 E_x}{\partial y^2} + \frac{g_3}{g_1^* g_2}\frac{\partial^2 E_y}{\partial x\partial y} \\ \gamma E_x = \frac{g_3^*}{g_1^* g_2}\frac{\partial^2 H_x}{\partial x\partial y} - \frac{g_3^*}{g_1 g_1^*}\frac{\partial^2 H_y}{\partial x^2} + g_3^* H_y + \frac{\partial E_x}{\partial z} \\ \gamma E_y = -g_3^* H_x + \frac{g_3^*}{g_2 g_2^*}\frac{\partial^2 H_x}{\partial y^2} - \frac{g_3^*}{g_1 g_2^*}\frac{\partial^2 H_y}{\partial x\partial y} + \frac{\partial E_y}{\partial z} \end{cases} \qquad (4)$$

where $g_i = j\omega\varepsilon+\sigma_i, g_i^* = j\omega\mu+\sigma_i^*; (i=x,y,z)$.

The problem analyzed in this paper is a periodic SIW structure synthesized with linear arrays of metallic via holes on a low-loss substrate covered by metallic walls. As shown in Figure 2, boundaries in the x direction are perfect electric conductors (PEC), and in the y direction are PML. In the z direction, we employ the Floquet theorem. In the frequency domain, equations in (4) can be discretized within a single period using a spatially interleaved Yee lattice [8]. Then, a generalized eigenvalue problem of such an open structure is derived

$$Ax=\gamma Bx \quad where\ A,B\in C^n \qquad (5)$$

In the periodic direction, when we make the number of lattices as an odd and move the components in the left-hand side of (4) by a half lattice, B will become a diagonal-block matrix, where each block is an invertible Toeplitz matrix. Hence, the inversion of matrix B can be determined easily, and then the generalized eigenvalue problem is simplified to a standard eigenvalue problem

$$B^{-1}Ax=\gamma x \qquad (6)$$

Therefore, the complex eigenvalues of the SIW propagation problem can be obtained by solving the matrix eigenvalue equation (6).

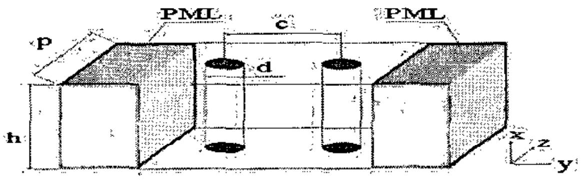

Fig. 2. Numerical model of the SIW synthesized using metallic via holes array in a single period.

III. Experimental Results

A. Experiment Procedure

In the experiment setup, the geometrical parameters of the SIW shown in Figure 1 are set as c = 7.112 mm, h = 2 mm, p = 1.5 mm, and d = 0.8mm. Figure 3 illustrates the connection of different experiment parts, in which Parts II_L and II_R are two rectangular waveguides tapering at one side of each with the size of 7.112 mm $\times$3.556 mm, and the middle part is either I_1 or I_2. Parts I_1 and I_2 are described as follows.

Part I_1: SIW, synthesized with linear arrays of metallic via holes on the low-loss substrate covered by metallic walls, as shown in Figure 1.

Part I_2: Rectangular waveguide filled with the same medium as the SIW substrate. The size of the waveguide is 7.112 mm$\times$2 mm.

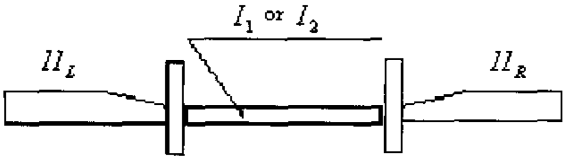

Fig. 3. The connection of different experiment parts.

The experiments are performed in the frequency range of 26.5-40GHz. Hence, only the dominant mode is the propagation mode in the above rectangular waveguides. In the experiment, Parts II_L and II_R are first connected directly and the corresponding S-parameters are measured. Then, Parts I_1 and I_2 are inserted between Parts II_L and II_R, respectively, and the corresponding S-parameters are measured.

B. Data Processing

As we know, there is an essential difference between an eigenvalue problem and a source-driven problem. In addition, it is rather difficult to ensure the symmetry and stability of the tested devices because of the fabrication and experiment-operation reasons. The above problems have more impact on the magnitude of S-parameters, but less influence on their phase. Hence, only the propagation constant β, i. e, the imaginary part of γ, can be derived from the measured S-parameter data. Also, the phase of the S-parameters from the measurement (ϕ') is always in the range of [$-\pi$, π], while the phase in the simulation (ϕ) may contain a phase difference of $2n\pi$. That means,

$$\begin{cases} \phi = l\beta & l > 0 \\ \phi = \phi' + 2n\pi & \phi' \in [-\pi, +\pi), n \in N \end{cases} \quad (7)$$

in which l is distance that the electromagnetic wave propagates (i.e., l is the length of Parts I_1 or I_2), and ϕ is the phase change when the wave propagates along the distance l. Hence, n must be determined.

Under the working state of the dominant mode, the following relation is satisfied [9]

$$n = \left[\frac{fl}{v} \sqrt{1 - (1/(2a_{equal} f \sqrt{\varepsilon\mu}))^2} \right] \quad (8)$$

where $v = 1/\sqrt{\varepsilon\mu}$, f is working frequency, [$\bullet$] is an operator making the value integer, $a_{equal} = v/(2f\sqrt{1-(\beta_{cal}/k)^2})$ is the equivalent wide-side dimension of SIW, and β_{cal} is the imaginary part of the complex eigenvalue computed.

C. Experiment Results and Their Comparison with Numerical Results

The measurement is carried out on ten frequency points, and the experiment results and their comparison with numerical results are illustrated in Tables I, Fig. 4 and Fig.5. Since the errors between the measured values and the theoretical results of Parts I_2 are all positive, we regard the average error of Part I_2 as the system error. After the system calibration, the corrected measurement results of the SIW are illustrated in Table I and Fig. 5, from which we clearly see that the numerical results have excellent agreement with the measurement data.

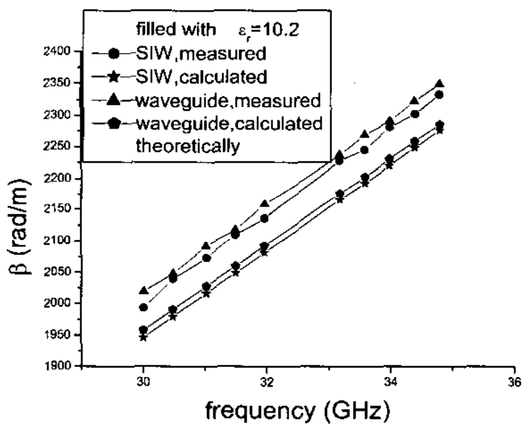

Fig. 4. Measured and simulation results before the system correction.

Table I

Measured and simulated results before and after the system correction.

frequency (GHz)	SIW (ε_r=10.2, $\tan\theta \le 10^{-3}$, θ is the loss angle of the medium)					β rectangular waveguide 7.112*2mm($\varepsilon_r = 10.2, \tan\theta \le 10^{-3}$)		
	$\gamma = \alpha + j\beta$ calculated	before system correction		After system correction				
		β measured	error(%)	β measured	error (%)	calculated	Measured	error (%)
30.0100	1.75e-4+1946.6i	1993.3	2.34	1936.7	-0.51	1958.2	2018.8	3.00
30.4825	1.68e-4+1979.2i	2038.8	2.92	1980.9	0.09	1990.5	2047.5	2.78
31.0225	1.61e-4+2016.4i	2073.2	2.74	2014.4	-0.10	2027.5	2091.9	3.08
31.4950	1.54e-4+2048.9i	2109.8	2.89	2049.9	0.05	2059.9	2117.8	2.74
31.9675	1.48e-4+2081.4i	2135.7	2.54	2075.1	-0.30	2092.2	2157.3	3.02
33.1825	1.30e-4+2164.8i	2228.2	2.85	2165.0	0.01	2175.2	2236.8	2.76
33.5875	1.22e-4+2192.5i	2245.1	2.34	2181.3	-0.51	2202.8	2268.6	2.90
33.9925	1.13e-4+2220.3i	2280.6	2.65	2215.9	-0.20	2230.4	2290.7	2.63
34.3975	1.02e-4+2248.0i	2302.1	2.35	2236.8	-0.50	2258.0	2322.7	2.78
34.8025	8.40e-5+2275.8i	2333.4	2.47	2267.2	-0.38	2285.6	2349.1	2.70

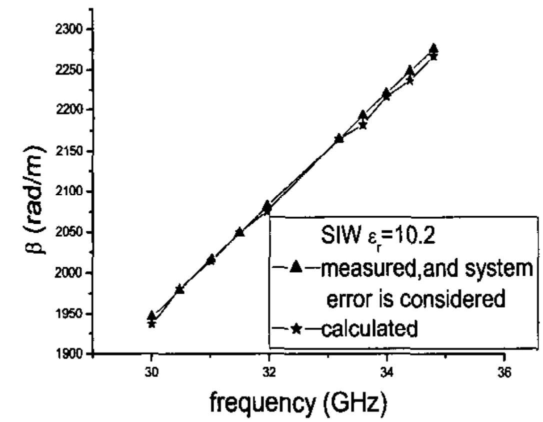

Fig.5. Measured and simulated results of the SIW structure after the system correction.

The number of solutions in solving (6) is equal to the rank of matrix $B^{-1}A$. The rules in selecting true solutions are omitted because of the paper length limitation.

All the possible modes have been computed and the false modes have also been extracted. In the real modes corresponding to the true complex roots of the eigen value equation, we find that some of them are very close to the higher-order modes of the rectangular waveguide which has the same geometrical size as the SIW structure. Since SIW and the rectangular waveguide have almost the same performance, it can be concluded that these complex roots are corresponding to the higher-order modes of SIW.

V. CONCLUSIONS

As a simple and an efficient method to analyze 3D periodic open structures, a novel FDFD algorithm is developed where the Floquet Theorem has been employed and PML has been chosen as ABC. Good agreements between numerical simulations and experimental results show the validity and efficiency of the proposed algorithm.

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